1 Introduction

During the last decade there has been an increasing interest in performability modeling [18]. The development of software tools supporting performability modeling and analysis has been an active area of research.

Metaphor [5], developed 1984, was the first tool for performability modeling. It addressed only a limited set of Markov models; input and output were textual. The tool SPNP [3] is a stochastic Petri nets package, which supports specification, generation, and solution of continuous time Markov chains. Steady-state, transient, and cumulative performability measures, defined by Markov reward models (MRMs), can be computed. The model description is done via C. The tool UltraSAN [4] is based on stochastic activity networks. In addition to numerical algorithms, UltraSAN provides also simulation methods. Surf-2 [1] has been developed for dependability and performability evaluation. Models can either be MRMs or generalized stochastic Petri nets. SPNP, UltraSAN and Surf-2 provide an output in tabular form. Additionally, Surf-2 allows a graphical representation of the results. Deterministic and general type time distributions are complementing exponential distribution in DSPNexpress [8] and in other work [2]. Many more tools do exist, most of them being covered in the overview paper by Haverkort and Niemegeers [6].

Figure 1: The friendly usable interface of PENELOPE.

This paper describes the new software package PENELOPE [13]. PENELOPE is the first tool which incorporates evaluation and optimization algorithms. It can be applied
for the integrated computation of performance/performability functions and of optimal control strategies. It constitutes the implementation of the concept of a combination between performability modeling and Markov decision theory [17].

In addition to transient and steady-state performability measures, also transient and steady-state control functions can be computed and graphically presented. Model specification and specification of transient or steady-state control structure can also be separately performed and deliberately combined with each other. This allows the immediate comparison of the impact of various control strategies for a given model on the resulting performance measures.

The specification of control strategies is built on the paradigm of reconfigurability [11, 12]. The intuition behind is, decisions must be made to reconfigure or not to reconfigure a system from one state to another in order to optimize a given performance/performability measure. The mapping of the reconfiguration options on internal model representations suitable for the optimization algorithms and the application of appropriate algorithms is hidden from the user.

PENELOPE provides a friendly usable interface, being indicated in Fig. 1, for model generation and experimentation. In particular, the creation of model variations is supported as well as the execution of series of experiments and the integrated presentation of the results of those experiments. This includes the presentation of performability functions and, in particular, the presentation of control strategies. No interference of the user is necessary to prepare the graphical presentation of control strategies and performability functions. The control functions are automatically related to the original specification of reconfigurability options and series of experiments. Thus, the execution of an optimization study can be considered as a meta-experiment that comprises many single experiments which are related to each other.

This paper is organized as follows. Section 2 contains a description of the general functionality of PENELOPE. Section 3 illustrates by means of a simple example important features of the tool, such as model generation, experiment set-up and execution, or presentation of results. Section 4 concludes the paper.

2 Description of PENELOPE

PENELOPE is based on the theory of extended Markov reward models (EMRMs) [11, 12]. It offers a modeling methodology that combines Markov reward models and Markov decision processes.

PENELOPE offers the following functionalities:

- Interactive graphical modeling.
- Automated checking of model consistency.
- Mechanism for hierarchical and iterative modeling.
- Definition of model parameters.
- Automated execution of series of experiments.
Graphical preparation of experimental results.

Interactive preparation of computed strategies.

Printing of models and results for documentation purposes.

Operating system independent data management.

The tool architecture is depicted in Fig. 2. Users communicate through a graphical user interface (gui) with five components: model component, parameter component, experiment component, result analysis component, and strategy component. Each of these has its own range of functional competence. The five user accessible modules communicate with each other and with a database management system (DBMS) according to the structure as indicated in Fig. 2. The DBMS is hidden from the user.

PENELOPE allows to create parameterized models of arbitrary finite size and to provide automatically the models with concrete values. To each parameter an arbitrary set of concrete values can be allocated. For each possible combination of parameter values, PENELOPE performs an experiment. Whole series of experiment can thus be easily specified and executed.

The mathematical background of EMRMs will be briefly presented in this section as far as it is necessary to introduce the analysis techniques of our tool. Let $Z = \{Z(t), t \geq 0\}$ denote a continuous time Markov chain with finite state space $S$. To each state $s \in S$ a real-valued reward rate $r(s)$, $r : S \rightarrow \mathbb{R}$, is assigned, such that if the Markov chain is in state $Z(t) \in S$ at time $t$, then the instantaneous reward rate of the Markov chain at time $t$ is defined as $X(t) = r_{Z(t)}$. In the time horizon $[0, t]$ the total reward $Y(t) = \int_0^t X(\tau) d\tau$ is accumulated. Note that $X(t)$ and $Y(t)$ depend on $Z(t)$ and on an initial state. The distribution function $\Psi(y, t) = P(Y(t) \leq y)$ is called the performability. For ergodic models
the instantaneous reward rate and the time averaged total reward converge in the limit to the same overall reward rate \( \lim_{t \to \infty} E[X(t)] = \lim_{t \to \infty} \frac{1}{t} E[Y(t)] = E[X] \).

For each unit of time in which the system is in a certain state, the corresponding number of reward units \( X(t) \) is added to the total reward \( \int_0^t X(\tau) d\tau \). Thus, it is possible to define a "yield measure" or a "loss measure" for the system by using appropriate reward rates.

EMRMs provide a framework for the combined evaluation and optimization of reconfigurable systems by introducing some new features for MRMs. A reconfiguration arc, which can be placed between an arbitrary Markov state and any other state in a model, specifies an optional, instantaneous state transition that can be controlled for optimization. Zero, one, or more reconfiguration arcs may originate from any Markov state. The resulting strategy, which provides optimal reconfiguration decisions for each option in the model, is commonly time-dependent. The so-called branching states provide another feature of EMRMs. No time is spent in such states, but a pulse reward may be associated with them. The introduction of branching states has motivation similar to the introduction of immediate transitions to stochastic Petri nets [9].

Two types of methods are offered for computation of optimal strategies and performance functions:

- **Transient optimization**, where the expected accumulated reward \( E[Y_i(t)] \) is used as an optimization criterion. The algorithm which has been introduced in earlier work [10, 11, 12] is applied for an analysis within a finite period of time \([0, t] \). Transient optimization for acyclic CTMC was introduced and its correctness proved by Lee & Shin [7]. The algorithm was adopted by de Meer [11] and extended to general CTMC. Additionally, the correctness of EMRM approach in terms of Markov decision processes [17] was proved. The algorithm is based on Taylor series1.

- **Stationary optimization**, which is performed for an infinite time horizon \([0, \infty) \). As optimization criteria, we distinguish between time averaged mean total reward in steady-state, \( E[X] = E[X_i] = \lim_{t \to \infty} \frac{1}{t} E[Y_i(t)] \) for all \( i \), where \( E[X_i] \) is independent of initial state \( i \) for a selected strategy, and the conditional accumulated reward until absorption, \( E[Y_i(\infty)] = \lim_{t \to \infty} E[Y_i(t)] \), which is computed for non-ergodic models containing an absorbing state. \( E[Y_i(\infty)] \) is dependent on initial state \( i \). The optimization itself is performed by deployment of variants of value iteration or strategy iteration type methods [11], relying on numerical algorithms such as Gaussian elimination, Gauss-Seidel iteration, successive over-relaxation (SOR), and the Power-Metho. All these methods are implemented in the tool and can be deliberately chosen for a computation.

In addition, PENELOPE offers procedures for computations under fixed deliberately eligible strategies:

- **Simulation**, where model evaluations can be simulated under fixed strategies.

- **Transient analysis**, where transient numerical evaluations can be carried out under fixed transient or stationary strategies.

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1Taylor series for transient analysis (without optimization) has also later been discussed by Stewart [15].
• **Stationary analysis**, where stationary numerical evaluations can be carried out under fixed stationary strategies.

The software tool has been built with the C programming language under the UNIX operating system, making use of X-windows and Motif.

## 3 A Simple Example

### 3.1 Description

The features of PENELOPE will be demonstrated by means of a simple example. In packet-switched networks, there exists the special case that applications, generating mixed traffic (data, video, voice etc.) with different quality-of-service requirements, are communicating via a single switching node. This is a typical situation in particular in a local-area environment, where the switch may be used as a PBX (Private Branch Exchange) [16].

One of the most important problems to be solved with respect to traffic management is related to congestion control. Congestion control for real-time traffic by selectively discarding packets has been investigated by Schulzrinne et al. [14]. We adopt a similar scenario in a simplified setting as depicted in Fig. 3. $n$ classes of packet streams are distinguished, where the classes differ from each other with respect to response time limits and loss thresholds.

Packets are continuously fed into the system according to independent Poisson processes with arrival rates $\lambda_1, \ldots, \lambda_n$. The packets are processed with rate $\mu$ using the service strategy *first come first served*. The problem of congestion control through discarding of last arriving packets will be investigated.

![Figure 3: A queueing system with $n$ classes of arriving packets.](image)

Keeping it as simple as possible, a system with $n = 2$ arrival streams will be considered. The different response time limits$^2$ of both classes are assumed to be proportional to the mean service time $\frac{1}{\mu}$ with different factors. The first class is *loss-tolerant* but more delay-sensitive. While having no limit for losses, the mean response time limit of these packets is assumed to be $2$ time units. There is no use in keeping class one packets if their expected response time is larger than that, that is, if there are already at least two packets in queue upon their arrival. The second class is highly loss-sensitive, no packet of this class may get lost. The less restrictive mean response time limit, on the other hand, is assumed to be $3$ units of time. Packets of the second class can be accepted as long as there are at most two packets in the queue ahead of them. Since loss of class two packets cannot be tolerated,$^2$ we define response time as waiting time plus service time.

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$^2$ We define response time as waiting time plus service time.
service is assumed to be immediately stopped if such an event occurs. The admission policies, optimizing different reward functions, will be studied in this simple example for purpose of demonstrating the features of the tool.

![EMRM Diagram](image)

**Figure 4:** The baseline model as an EMRM.

The baseline model is depicted as an EMRM in Fig. 4. In states $N_j$, $j \in \{0, 1, 2, 3\}$, $j$ packets are in the system. In states $R_i$, $i \in \{1, 2\}$, $i$ packets are in the system and the last arriving packet is a packet of the first class. Whenever the system is in state $R_i$, a decision has to be made whether the last arriving packet should be dropped or not. In case of a positive decision the system is reconfigured to the corresponding state $N_{i-1}$. The reconfiguration options are graphically indicated by dashed lines in the EMRM of Fig. 4. If loss of a second class packet occurs, then the absorbing state $A$ is reached and all further arriving packets, regardless of their class, are considered to be lost. First, we will investigate the strategy, which maximizes the mean throughput in terms of the number of packets served in finite time or before service interruption occurs. Then, the strategy which minimizes the expected number of lost packets and differences in the strategies will be discussed.

![Model Editor](image)

**Figure 5:** Model editor of PENELOPE.

In **PENELOPE** the resulting model can be easily created with the help of the graphical model editor as illustrated by Fig. 5, which is provided as part of the user interface.

The expected accumulated reward, as a measure of accomplishment, is used as a criterion of optimization. Series of experiments for different reward structures will be investigated. It
is interesting to compare the impact of various control strategies on the resulting performance measures. As it will turn out, transient strategies yield better performance results than stationary strategies do. In addition, the difference will be investigated between using the mean accumulated reward in finite time as an optimization criterion and applying the mean accumulated reward until absorption as a criterion.

3.2 Experiments

3.2.1 Maximization of Throughput

To specify the throughput per unit of time, the reward rate is defined as the service rate times the number of active processors in each state. The resulting reward structure is attached to Fig. 8. The set of parameters is summarized in Tab. 1. Users can define parameter values either directly in the model editor or in the parameter set editor, which allows one to explicitly specify an arbitrary set of concrete values to be substituted for formal parameters. One can equivalently specify parameter value-ranges and step widths, where for each of the resulting values a computation will automatically be executed. Usage of the parameter set editor is exemplified by Fig. 6.

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
<th>meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_1$</td>
<td>0.02</td>
<td>arrival rate of the class 1</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>0.02</td>
<td>arrival rate of the class 2</td>
</tr>
<tr>
<td>$\mu$</td>
<td>$[0.01, \ldots, 0.13]$</td>
<td>service rate</td>
</tr>
</tbody>
</table>

Table 1: Set of parameters.

Figure 6: Parameter set editor.

Using the experiment editor, which is shown in Fig. 7, complete series of experiments can be specified and executed.
Transient Optimization  In our example transient optimization is performed in the time horizon $[0,6000)$. Assuming one unit of time corresponding to 10 msec, the total arrival rate $\lambda_1 + \lambda_2$ results in 1 packet/250 msec $= 4$/sec. The time horizon would cover 1 min. The resulting dynamic control strategy, which maximizes the throughput, is depicted in Fig. 8. With respect to state $R_2$, the two-dimensional decision space, given by the covered set of parameter values of $\mu$ and the considered time interval, is partitioned into two regions, which are divided by the curve with the label $'R_2 \rightarrow N_1'$ attached to it. In the region above the curve the indicated strategy applies: A message of class one should be discarded in favor of potentially arriving class two messages. If, however, conditions are given such that the current situation is classified to be in the region underneath the curve, the alternative strategy should be applied in state $R_2$ and class one message should not be discarded in order to maximize the overall throughput. The region is indicated by label $'R_2 \rightarrow R_2'$.

With respect to state $R_1$, the two-dimensional decision space comprises a single region. Neither time nor service rate affect the curve labeled $'R_1 \rightarrow R_1'$, This means that the strategy ‘do not discard the last arriving packet in state $R_1$’ is applied in the whole decision space.

The curves, also referred to as switching curves, represent for each state the instants of time where the strategy switches with respect to the remaining time. For example, if the service rate is $\mu = 0.08$, that is, 1 packet/125 msec, and if the time to go is 500 units, that is, 5 sec, an arriving packet of the first class should be accepted. The point $(0.08, 500)$ is located in the region $R_2 \rightarrow N_1$ and $R_1 \rightarrow R_1$. The throughput would decrease by dropping a packet in such a situation. The strategy reveals the following monotony: the smaller the service rate, the less packets of the first class are accepted. Of course, the loss risk of a second class message is higher with a decreasing service rate.

In Fig. 9, the throughput for the initial states $N_0$, $N_1$, $N_2$, and $N_3$ is depicted as a function of the service rate. The reward functions are computed assuming the optimal strategy. The throughput behaves in a way which might have been anticipated: the higher the service
rate, the higher the throughput. In the right part of Fig. 9, it is to be seen that the system reaches the highest throughput with the initial state $N_1$. Note that in PENELLOPE one can arbitrarily select strategy/performance curves to be graphically presented. In Fig. 9, four curves were selected.

In what follows, the impact of arrival rates $\lambda_1$ and $\lambda_2$ on the optimal control strategy will be investigated, while $\mu = 0.04$ will be kept fixed. For the computations relating to the left part of Fig. 10, the arrival rate $\lambda_1$ of class one messages is varied, while $\lambda_2 = 0.02$ is kept fixed. In the right part, the arrival rate $\lambda_2$ of the second class messages is varied, while $\lambda_1 = 0.02$ is kept fixed. It is interesting to note that the strategy switching curves decrease with increasing arrival rates in both cases. But the strategy as a function of $\lambda_2$ is much more sensitive to an increase of the parameter value than in the first case. If $\lambda_2$ is significantly larger than $\lambda_1$, the option to discard class one message in state $R_2$ is "nearly always" selected. Note that in state $R_1$ a message should never be discarded.

<table>
<thead>
<tr>
<th>state</th>
<th>reward rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_0$</td>
<td>0</td>
</tr>
<tr>
<td>$N_i$</td>
<td>$\mu$</td>
</tr>
<tr>
<td>$R_j$</td>
<td>$\mu$</td>
</tr>
<tr>
<td>$A$</td>
<td>0</td>
</tr>
</tbody>
</table>

$i \in \{1, 2, 3\}, j \in \{1, 2\}$

Table 2: Optimal stationary strategy.

**Stationary Optimization** The stationary optimization is performed for the same parameter set as in Tab. 1. The strategy iteration with Gauss-Seidel computation method is chosen due to the small size of the model. The accumulated reward until absorption is used as a criterion of optimization. The resulting strategy is summarized in Tab. 2. It applies for
the whole interval of considered service rates. If one compares the stationary strategies from Tab. 2 with the transient strategies in Fig. 8, it can be seen how they relate to each other. In the long run messages should always be discarded in state $R_2$ and never in state $R_1$. But in state $R_2$ the optimal strategy is strongly time-dependent for shorter periods of time.

**Transient Analysis**  PENELOPE allows to save and modify computed strategies, or to create arbitrary new strategies, and to apply them in computations. Using this feature, it is possible to execute series of experiments under different strategies and to compare their impact on performance. To compare, for example, the impact of optimal transient and optimal stationary strategies on the performance in our scenario, a transient analysis under the optimal stationary strategy from Tab. 2 is performed.

For easy comparison of the results, we use another feature of PENELOPE: the provisioning of difference graphs, which can be derived by combining arbitrary experiments. In Fig. 11, a difference graph for states $N_0$, $N_2$, and $N_3$ is depicted. Each point of this graph is a result of a subtraction $a - b$, where $a$ is the throughput computed under the optimal transient strategy, as depicted in Fig. 8, and $b$ is the throughput computed under the optimal stationary strategy from Tab. 2 for corresponding service rates. As is to be seen from Fig. 11, no difference in performance can be observed for service rates $\mu < 0.04$, that is, if $\frac{1}{\mu} + \frac{1}{\bar{\mu}} > 1$. The relative high load causes the system to reach the absorbing state quickly, regardless of the type of optimization being applied. Therefore, the mean accumulated throughput of the system in the time horizon $[0, 6000]$ is equal to the mean throughput until absorption in both cases. If the service rate becomes sufficiently large, $\mu > 0.04$, the difference strongly increases. In other words, the higher the service rate, the better the resulting performance becomes when adopting the optimal transient strategy as opposed to the optimal stationary strategy.
Figure 10: Dynamic control strategies as a function of time and of arrival rates.

Simulation  PENELPOE also provides simulation as a method of evaluation. Two simulation studies are performed for the initial state $N3$ and time horizon $[0,6000]$ under the optimal transient strategy, as depicted in Fig. 8: First, with 100 runs; second, with 1000 runs. The mean accumulated throughput is depicted in Fig. 11(right). The curves are labeled with the corresponding number of runs in parentheses. The curve labeled 'N3' is the mean accumulated throughput computed with transient analysis assuming the same strategy. Our tool allows to combine arbitrary strategy/performance curves with each other, which is helpful for comparison of results from different series of experiments. We used this feature for creating the graph in Fig. 11(right).

PENELPOE offers facilities for tracing performance functions, which depend on some initial states, in an arbitrary time interval. With the help of this feature the mean accumulated reward is depicted as a function of time. Fig. 12 compares the mean accumulated throughput for the initial state $N1$ and service rate $\mu = 0.08$ computed under different strategies. The curve labeled "sopt" indicates the mean accumulated reward until absorption computed with the stationary optimization. The curve labeled "tana" is a result of the transient analysis under the optimal stationary strategy shown in Tab. 2. The curve labeled "topt" is a result of the transient analysis under the optimal transient strategy shown in Fig. 8. From the right part of Fig. 12 it can be concluded that optimal transient strategies yield better results than optimal stationary strategies.

3.2.2 Minimization of Losses

The optimal strategy, which minimizes the mean number of lost class one packets, will be investigated. In order to do that, we have to modify the model from Fig. 4 slightly. In Fig. 13 branching states $I0$, $I1$, and $I2$ were added. The new resulting reward structure is summarized in Tab. 3. We assign the reward (cost) rate $\lambda_1$ to states in which arriving class
Figure 11: Difference graph of throughput computed under different strategies (left) and mean accumulated throughput for initial state $N3$ computed with simulation and tran. analysis (right).

<table>
<thead>
<tr>
<th>state</th>
<th>reward rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_i$</td>
<td>0</td>
</tr>
<tr>
<td>$N_j$</td>
<td>$\lambda_i$</td>
</tr>
<tr>
<td>$R_1$</td>
<td>0</td>
</tr>
<tr>
<td>$R_2$</td>
<td>$\lambda_1$</td>
</tr>
<tr>
<td>$I_i$</td>
<td>1</td>
</tr>
<tr>
<td>$I_2$</td>
<td>$2p_1p_2 + p_1^2$</td>
</tr>
<tr>
<td>$A$</td>
<td>$\lambda_1$</td>
</tr>
</tbody>
</table>

Table 3: Reward rates for minimization of lost class one packets.

one packets get lost, that is, if there are at least two messages ahead, or if the absorbing state has already been reached. Furthermore, if a reconfiguration is executed from state $R_i$ to state $I_{i-1}$, $i \in \{1, 2\}$, one packet of the first class is dropped. This is captured by pulse rewards $1$, that are assigned to branching states $I0$ and $I1$. The pulse reward assigned to state $I2$ corresponds to the expected number of lost class one packets, being in the system at the moment of service disruption. Note that a minimization of expected accumulated rewards can be realized in PENELOPE by specifying negative reward rates.

The optimal transient strategy is depicted in the left part of Fig. 14. If the system is in the state $R_1$ then the strategy is the same as the one of the throughput model: Packets of class one are accepted for the whole time horizon and for all values of the considered service rate parameter interval. If the system is in the state $R_2$, however, two overlapping effects are observed.

If the service rate is relatively small, or equivalently, the load relatively high, it can be observed that the smaller the service rates the less it pays off to discard a packet. This is due to the fact that the absorbing state will be reached very quickly under that load condition.
For service rates $\mu < 0.04$, packets will therefore never be discarded if the total loss is to be minimized.

A reverse effect dominates for larger values of $\mu$: The higher the service rate, the less the expensive discarding is performed. The risk of reaching the absorbing state decreases with an increasing service rate. It becomes more and more likely that the queue will shrink again before next arrival of a class two message. The overlapping of reverse effects results in a minimum of the switching curve $R_2 \rightarrow I_1$, which partitions the decision space into regions, where alternative strategies apply.

The right part of Fig. 14 shows the total loss in time horizon $[0, 6000)$ as a function of the service rate for the initial states $N_0, N_1, N_2$, and $N_3$. With increasing service rate the mean accumulated reward decreases. It is also evident, that the less packets are initially in the queue, the less packets get lost.

It is also interesting to investigate the impact of the transient control strategy, minimizing the total loss, on the performance in terms of mean accumulated throughput. We refer back to the parameter set summarized in Tab. 1 and select again the time horizon $[0, 6000)$ for transient analysis. With the throughput model (Fig. 4) and the attached reward structure from Fig. 8, a transient analysis under the strategy from Fig. 14, which minimizes losses, is performed. Since this strategy is not optimal for the throughput model, the resulting
Figure 14: Strategy for minimization of lost packets (left) and mean total loss under this optimal strategy (right).

Performance is poorer than the one in Fig. 9, gained under the optimal strategy. The difference graph is depicted in the left part of Fig. 15. Each point is a result of a subtraction $a - b$, where $a$ is the throughput computed under the transient strategy, which is optimal for the throughput model (Fig. 8), and $b$ is the throughput computed under the transient strategy, which is optimal for the loss model (Fig. 14).

The right part of Fig. 15 shows results from a reverse experiment. The baseline model is the model from Fig. 13, with the reward structure from Tab. 3 attached. $a$ is the mean accumulated loss computed under the transient strategy, which is optimal for the loss model (Fig. 14), and $b$ is the mean accumulated loss computed under the transient strategy optimal for the throughput model (Fig. 8).
Figure 15: Difference graph of throughput and losses computed under different strategies.

4 Conclusions

We have presented a new modeling tool which can be used for the combined optimization and evaluation of reconfigurable systems. The important features of PENELLOPE have been introduced by means of example. In particular, it was demonstrated how transient optimization techniques and optimization until absorption can be applied for congestion control problems by providing selective packet discarding strategies. It was shown how model structures and optimization strategies could be easily combined with each other in order to compare the impact of different control strategies on resulting performance or performability functions. While series of experiments being flexibly accomplished with a minimum of user interference, complex control strategies can be presented in an abstract way directly relating to a series of input models. The specification of control options is based on the paradigm of reconfigurability. A knowledge of details of the underlying algorithms is therefore not necessary in order to apply optimization techniques.

For the sake of completeness we may mention that the largest optimization models which were investigated with our tool had a size in the order of some 100,000 states.

Continuing effort is made to improve the interface further and to extend the class of implemented numerical algorithms.
References


